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A Comparison of Pseudo-Noise and Conventional Modulation for Multiple-Access Satellite Communications*

Abstract: This paper compares pseudo-noise (PN) with conventional modulation techniques for multiple-access satellite communications of voice messages. The reference for all comparisons is the conventional frequency-division multiplex telephone system. The comparison study is concerned with theoretical channel parameters as well as practical considerations which are unique to satellite communications. For PN modulation, curves are presented which show the relationship between the intrinsic signal-to-noise ratio and the number of channels per megacycle for a given test-tone-to-noise ratio.

It is concluded that high quality voice transmission can be achieved efficiently with PN-multiplexing. In particular, pulsed pseudo-noise transmission with some form of wide-deviation pulse-time message modulation and matched filter reception uses the down-link intrinsic signal-to-noise ratio and bandwidth as efficiently as the conventional single-sideband up and composite frequency-modulation with feedback down, provided that up-link power control is used. For lower quality communications, conventional modulation is more efficient in rf bandwidth utilization. Where rf bandwidth is not a significant factor, but the down-link intrinsic signal-to-noise ratio is important, then in the case of PN modulation, communications can be made thermal noise limited in the down-link. Here PN is, for all practical purposes, as efficient as orthogonal multiplexing.

Introduction

The goal of this paper is the theoretical comparison of pseudo-noise modulation** with conventional modulation techniques for multiple-access satellite communications. The fundamental work of Stewart and Huber[1] on analytical comparison of modulation techniques serves as the basis for the present study. Their methods have been used to extend the theory to include pseudo-noise techniques. In particular, the theory is extended to digital communications using higher-order (*M*-ary) signal alphabets with pseudo-noise multiplexing, and to analog modulation systems multiplexed by pseudo-noise signals.

In a communication satellite system many ground stations have access to the satellite simultaneously; hence, the term "multiple access." The satellite serves as a multiplexing point for the received signals. Since it is impractical to place a switching central in the satellite, techniques of communications must be devised which perform this function.

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** The term "pseudo-noise modulation" refers to a type of modulation in which the phase of the rf carrier signal is changed in accordance with the output signal of a maximum-length (i.e., pseudo-noise) sequence generator. The book by Golomb, et al.[2] gives a number of basic works on pseudo-noise signaling.

The simplest communication satellite is a repeater, which receives transmitted signals in one region of the radio spectrum and retransmits them in another. Telstar, Relay, and Syncom are examples of such communication satellite repeaters which have been successfully tested. This paper assumes that the communications link contains a repeater satellite.

Because of payload limitations, the communication satellite is severely power limited and requires modulation schemes which make extremely efficient use of the on-board transmitter power. In addition, the peak-power-limited traveling-wave tube, which is aboard the satellite and is used for repeating the received signals, delivers maximum power when operating in its nonlinear saturation region. Because of these limitations, it is expected that the satellite system will accommodate more voice channels with some modulation techniques than with others for a given voice quality per channel. This paper addresses itself to this problem, calculating the relationship between the quality of a voice channel and the communication channel parameters for digital and analog PN modulation schemes. The PN techniques are then compared with each other as well as with conventional schemes. The important parameters in this comparison study are the voice quality (defined by a "test tone-to-noise ratio"), the signal-to-noise ratio in an equivalent K-channel single-sideband, frequency-division carrier system, and the number of channels per megacycle of rf bandwidth.

The optimum conventional modulation technique has been shown to be single-sideband, frequency-division multiplexing (SSB-FDM) in the up-link and composite frequency modulation with feedback (FMFB) in the down-link. We will show that the optimum PN system uses M-ary signal alphabets. Both FMFB and analog PN systems exchange on-board power for rf bandwidth, a necessary trade-off where high-quality voice reception is required.

For "high-quality voice" (i.e., test tone-to-noise ratio of 48 dB), the conventional and PN systems require approximately the same power and bandwidth. Where poorer quality is acceptable, the conventional techniques require substantially less rf bandwidth than pseudo-noise for the same power. On the other hand, where on-board power is expensive and rf bandwidth is cheap, pseudo-noise modulation using M-ary alphabets requires the minimum threshold power since the interference from other channels (clutter) can be made negligible. In this case, performance is shown to be thermal-noise limited, just as for orthogonal multiplexing. Finally, for PN to be competitive, it is essential to make full use of the voice channel activity factor. To accomplish this, voice-actuated rf carrier control at the ground should be used.

- *Satellite communications*

This study was performed with a synchronous satellite (e.g., Syncom) in mind, although the results are applicable to other repeater satellites as well. The satellite is a multiplexing point for the ground stations. The on-board power is shared by the signals received in the satellite. If the repeater is linear, the share of the on-board power taken by a ground station is proportional to the up-link power received from that station. A linear repeater would contain a slow-acting AGC to control the signal level so as to obtain maximum transmitter power efficiency from the on-board traveling-wave tube. In the case of pseudo-noise signaling, the repeater can contain a hard limiter, i.e., a highly non-linear signal-level control device. For some other modulation systems, the repeater must be operated in the linear region to reduce intermodulation.

The major problem in designing a satellite communications system is to provide the capability of accommodating strong and weak ground stations simultaneously. Even when a linear repeater is used, the larger station takes a proportionately larger share of the down-link power, thereby degrading the performance of the smaller stations. In order to eliminate this undesirable situation, either up-link power control, or time-division multiplexing (TDM) must be used. In the pseudo-noise systems considered here, we will assume that up-link power control is used. In a system like Syncom, the range is known accurately and the transmitter power can be controlled accurately and automatically, albeit at the expense of added complexity at the ground terminal. However, weather conditions can change the propagation conditions and hence, power received by the satellite. Nevertheless, it is possible to monitor the propagation medium, thereby deriving power correction information.

We will assume throughout that power control will be used in all the modulation systems considered except for the pulse-code-modulation, time-division multiplexing system (PCM-TDM), where precise time control is required. The importance of this systems problem is recognized; however, detailed consideration of it is beyond the scope of this paper.

- *Pseudo-noise modulation*

Fundamental to pseudo-noise modulation is the concept of bandwidth spreading—the transmission of signals whose rf bandwidth is much greater than the message bandwidth. Interference from signals using the same bandwidth is suppressed at the output of the correlation receiver by the bandwidth-time product (i.e., the signal-processing gain) of the pseudo-noise signal. This "redundancy" property of the signal permits many signals to share a common, broad band.

The pseudo-noise signals discussed in this paper are binary sequences which modulate the rf carrier by reversing its phase. However, the theory is applicable as well to an rf carrier angle-modulated by an arbitrary PN signal and recovered by a correlation receiver. Examples of systems using PN signals are given in the following paragraphs.

Figure 1 shows the transmitter and receiver of a pseudo-noise, frequency-modulation-with-feedback system (PN-FMFB)—an analog communication system. This system clearly uses a conventional FM signal which is spread over a much broader band by phase reversing in a known pattern generated by the pseudo-noise generator. The PN signal at the receiver is a replica of the transmitted signal and phase locked to it. Multiplication of the PN reference into the received signal removes the PN signal, thereby recovering the much narrower band conventional FM signal, which is then filtered and demodulated in an FMFB receiver.

Figure 2 shows the transmitter and receiver of a PN system where digital information is transmitted via an orthogonal M-ary alphabet; that is, one of a set of M orthogonal PN waves is transmitted for each message sequence. Here too, the PN signal has a bandwidth which is much greater than that of the M-ary alphabet signals. The desired signal is extracted from the received mixture

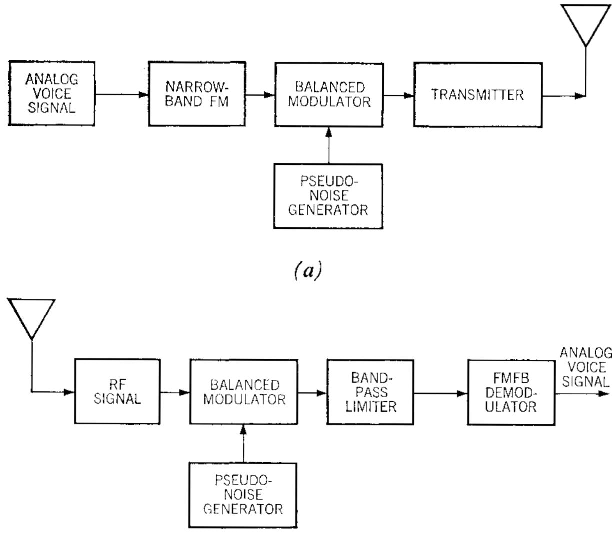
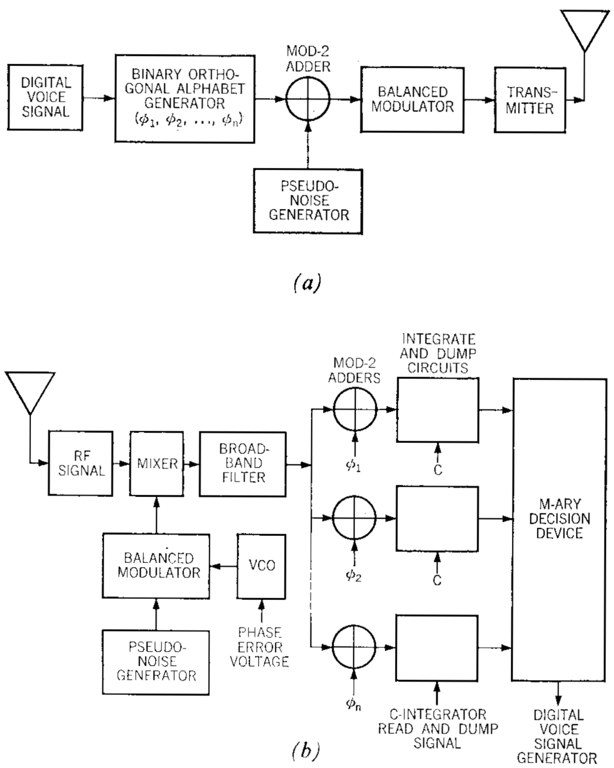


Figure 1 Pseudo-noise, frequency-modulation-with-feedback system. (a) Transmitter; (b) Receiver.

Figure 2 Pseudo-noise system using orthogonal binary codes. (a) Transmitter, (b) Receiver.



by means of a correlator that is synchronized with the received signals.

Corr, et al.[3] in this issue, describe another important implementation of a pseudo-noise communication system where the correlation receiver is a digital matched filter. Here the message is converted into quantized pulse-position modulation (PPM) and subsequently encoded into a pseudo-noise burst signal. The matched filter reconstructs the original PPM wave train. Such a system has the important property that message reception can be accomplished asynchronously, quasi-synchronously or, if desired, synchronously. An equivalent implementation can be realized by using analog PPM and an analog matched filter.

• *Conventional modulation techniques*

Stewart and Huber's comparison of several conventional modulation techniques is also presented for reference in this study: single-carrier PCM (i.e., PCM-TDM), single-carrier FM, multiple-carrier FM, and SSB-FDM.

PCM-TDM requires synchronization of all the ground as SSB-FDM which requires a linear repeater. In the satellite in an unoccupied time slot. Here, the repeater can be nonlinear.

In single-carrier FM, the up-link signals are transmitted as SSB-FDM which requires a linear repeater. In the satellite, the composite signal from all the ground stations frequency modulates a single rf carrier which is transmitted down. Each receiver demodulates the composite signal in a frequency detector either with or without feedback. The latter has a lower threshold and is therefore preferable. Each ground station selects its signals from the composite by means of an SSB frequency demultiplexor.

In multiple carrier systems each ground station is assigned a different receiving or transmitting rf bandwidth. Either a single receiver or multiple receivers are required to select the proper band. The links here must be linear in order to reduce intermodulation distortion.

• *Fidelity criterion*

A mathematical theory of modulation must contain a simple, meaningful, and reasonable fidelity criterion which describes system performance. A useful fidelity criterion in voice communication systems, both digital and analog, is the ratio of the power in the desired signal component and the mean-squared error between the desired signal and the demodulated message. The latter is the audio noise in the system. In the case of a digital system, there are two components of noise: quantization (thermal) noise and decision error noise.

• *Telephone system load factors*

When designing a radio telephone system, it is essential to take account of the voice load factors to ensure a prescribed performance. The composite signal at the output of an SSB-FDM voice system has characteristics which depend on individual voice signal statistics, individual talker volume, channel activity, pauses, etc. The first-order statistical model for the composite voice signal uses a Gaussian probability distribution function which expresses the percentage of time that a certain level is exceeded. This model has been developed by Holbrook and Dixon[4] after extensive experimentation over typical telephone systems. It is therefore an empirical model which has been found useful for designing practical voice systems.

For design analysis and testing purposes, the actual voice signal is replaced by an equivalent sinusoid whose peak value is exceeded by the voice signal a specified percentage of the time (for example, 1%). This is called the full-load sinusoid. It is numerically specified by the ratio of its rms value to that of a 1 mW standard test tone at the point of zero transmission level. When so expressed it is called the rms load factor.

Mathematical assumptions

- *Clutter process*

We assume that clutter, the mixture of equal power interfering signals using the common frequency band, is a random process at the input to the correlator and a Gaussian process at the output of the correlation receiver. The random process assumption comes about in part from the fact that, in the PN systems considered, a different collection of interfering signals overlaps the desired signal during each integration period. In addition, the large bandwidth-time product of the signals brings the "central limit theorem" into play since the output of the correlator consists of the sum of many independent signal components having the same probability distribution. Hence, the clutter process tends toward a white (flat spectrum) Gaussian process and the calculations are expected to be quite representative. Limited experiments over satellite links and computer simulation[5] have demonstrated the validity of the mathematical model used to compute the correlation output signal-to-noise ratio.

- *Error probability*

For the large correlator output signal-to-noise ratios required to achieve high-quality voice, the simplified error probability expressions used here are very good approximations to the exact mathematical expressions which are much more complex.[6,7,8] For M-ary alphabets where $M \geq 32$, the approximate expression used for matched filter reception with a post-envelope detection decision procedure is for all practical purposes the same as for the case of pre-envelope detection decisions (i.e., rf-coherent integration). In any case, the expression which we use is a good upper bound for the rf-coherent correlation receiver and our results are therefore conservative in the latter case.

- *Audio signal-to-noise ratio*

The fidelity criterion used assumes a flat probability density function of the voice sample values. This assumption is a very good lower bound for high-quality voice for any reasonable set of signal statistics.[4,9] Since quantized sinusoid signals give an audio signal-to-noise ratio 3/2 as large as the one used here, our results are useful and practical as well as being conservative. Furthermore, in PN systems we use the same fidelity criterion for M-ary detection as for binary detection for the sake of simplicity, recognizing that only a very small difference exists in the region of practical interest.

In the case of multi-word decisions, we also use an approximation for the fidelity factor, which, in the region of interest, differs by a negligible amount from the exact expression.

- *Activity factor*

A voice signal has gaps representing pauses in the conversation. In order to make optimum use of the channel, the rf carrier should be turned off during these gaps reducing the average clutter in the common band.[10] Tests on telephone circuits have shown that an activity factor of 25% is a good assumption for a system having many voice channels. We assume that the mixture of interrupted clutter signals forms a continuous random process having an average power proportional to the activity factor.

- *Threshold characteristic*

In the model of the digital and analog detection systems using pseudo-noise, as we have mentioned, there are two competing audio noises: decision noise and quantization noise. From a mathematical point of view, an attempt to decrease one form of the noise necessarily increases the other for the same correlation output signal-to-noise ratio.[11] This complementary behavior implies that the audio signal-to-noise ratio can be optimized with respect to the alphabet size in the case of digital communication and with respect to modulation index in FM. An excellent approximation to this optimum is obtained by equating the two competing noise powers. In this manner, the "knee" (i.e., threshold) in the fidelity characteristic is calculated.

- *Load factor*

In the case of PN signals, we assume that one voice channel is transmitted on a PN carrier. Hence, the single-channel rms load factor of 9.5 dB is used and subtracted from the theoretical audio signal-to-noise ratio, giving the expression for test-tone-to-noise $(T.T./N)$.[4,12] In addition, a 3.5 dB noise improvement due to "psophometric weighting"* is included.[10]

The subjective advantages of companding, an important operation in voice systems, are not included in these results. These empirical factors are available[1] and can always be added to our results.

Analysis of digital PN multiplexing systems

In this section, we develop the relationships that are necessary to allow pseudo-noise multiplexing systems using quantized sampled data (i.e., digital) modulation techniques to be expressed in the terms of a conventional reference system. The chosen reference system is the single-sideband, frequency-division multiplexed system. Three reasons for the selection of the SSB-FDM system as a reference are as follows: (1) It makes the most efficient use of channel bandwidth and therefore maximizes the number of channels per megacycle; (2) It has been used in the past for the comparison of conventional modulation techniques; and (3) It is the best understood voice communications system in existence.

The comparison method[1] used here is one in which single voice-channel characteristics are specified in terms of the test tone-to-noise ratio $(T.T./N)$ at the point of zero transmission level. Thus, in this analysis, we first derive an expression for the audio signal-to-noise ratio (S/N) of a communications system in which the audio signal is sampled and quantized. We then express $T.T./N$ in terms of S/N, and finally express $T.T./N$ as a function of the communications channel parameters of the reference SSB-FDM system.

- *Audio signal-to-noise ratio*

An important measure of system performance is the ratio of mean square signal to mean square error. If S_n is the output at the nth sample time in the absence of noise and O_n is the actual nth output sample, the error in the nth sample will be $\epsilon_n = S_n - O_n$. The mean square signal to mean square error ratio will then become,

$$\frac{\langle S_m^2\rangle}{\langle \epsilon_m^2\rangle} = \frac{\lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_{n=1}^{N} S_n^2}{\lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_{n=1}^{N} \epsilon_n^2}. \tag{1}$$

This fidelity criterion will be defined as S/N.

In digital voice transmission systems such as described here, the audio signal is sampled and quantized into M values. The M values are then transmitted as one of M possible waveforms. At the receiver, the waveforms are detected and the audio signal is retrieved by appropriate low pass filtering.

* Psophometric weighting is a term used to refer to the frequency response characteristics of the human ear.

The audio signal-to-noise ratio will depend upon the number of quantization levels, M, and the probability of incorrect detection, α. This probability will in turn depend upon the signal-to-noise ratio at the output of the correlation receiver and on the method of detection. (The noise here consists of thermal noise plus clutter.)

First, an expression for S/N as a function of M and α will be given. Although the method of detection may drastically change the value of α, for all practical purposes the relationship for audio signal-to-noise remains unchanged.

A mathematically convenient model for the statistics of the analog source is that the amplitude probability density is flat.[11] This model can be implemented by using a compander.

In Ref. 11 it is shown that the audio output signal-to-noise ratio has the functional form,

$$\left(\frac{S}{N}\right) = \frac{M^2 - 1}{1 + 4\alpha_B(M^2 - 1)}, \tag{2}$$

where M is the number of levels which partition the range $(-A, A)$ and α_B is the bit error rate. It is now essential to calculate the bit error probability α_B for various methods of transmission. In the M-ary case it is shown[11] that,

$$\alpha_B = \frac{M}{2(M-1)}\alpha, \tag{3}$$

where α is the M-ary decision probability of error. From Eqs. (2) and (3)

$$\left(\frac{S}{N}\right) = \frac{(M^2 - 1)}{1 + 2M(M+1)\alpha}$$

$$\cong \frac{1}{2\alpha + M^{-2}} \qquad \text{for} \qquad M \gg 1. \tag{4}$$

When bit-by-bit decision is used (i.e., as in conventional PCM), for high quality voice we have from Eq. (2)

$$\left(\frac{S}{N}\right) = \frac{1}{4\alpha + M^{-2}} \tag{5}$$

where $\alpha = \alpha_B$, the bit error probability.

In this paper, we also consider multiple word decisions. This type of decision produces a small modification of the expression for S/N. For large values of M, i.e., for good voice quality, the factors which multiply the error probability will influence the fidelity expression S/N only slightly.[9] Thus, for the purpose of this study, we use Eq. (4) for all the decision procedures studied, recognizing that for $M \gg 1$ the discrepancy between the exact expression and the one used is of little practical significance.

Before introducing the concept of $T.T./N$, it will be useful to express S/N as given in Eq. (4) as a function of η^2, the average signal-to-noise ratio at the correlation receiver output (η^2 is related to the peak signal-to-noise ratio by $\eta^2 = \frac{1}{2}\eta_p^2$). The probability of error α is related to the signal-to-noise ratio at the output of a matched filter that makes a "greatest-of" decision by the equation,[6,7,8]

$$\alpha = \frac{M-1}{2} \exp\left(-\tfrac{1}{2}\eta^2\right), \tag{6}$$

where an orthogonal signal alphabet, envelope detection, and $\eta^2 \gg 1$ are assumed. (This equation is a very good upper bound on the phase-coherent case as well, when $M \gg 1$). Substitution of Eq. (6) into Eq. (4) and expressing the results in decibels yields,

$$\left(\frac{S}{N}\right)_{dB} = 2.2\eta^2 - 10 \log_{10}(M-1) - 10 \log_{10}\left[1 + \frac{Q(M)}{M-1} \exp \tfrac{1}{2}\eta^2\right], \tag{7}$$

where $Q(M)$ represents the quantization noise-to-signal ratio. In general,

$$Q(M) = 2^{-2m_b} \tag{8}$$

where m_b = total number of bits in the sample.

It is now necessary to extend the results to the case where several word decisions are made in order to detect the sample value. If q words of m bits each are used, then

$$m_b = mq. \tag{9}$$

For a sampling period $T_s = 1/2W_0$ and a word decision time duration T,

$$T = T_s/q = 1/2W_0q, \tag{10}$$

where W_0 is the audio bandwidth. Therefore,

$$q = 1/2W_0T. \tag{11}$$

Both $Q(M)$ and η^2 are functions of q. The quantization noise-to-signal ratio is given by

$$Q(M) = 2^{-2m_b} = (2^{-m})^{2q} = M^{-2q}. \tag{12}$$

For the case of word decisions, $q = 1$ and Eq. (12) reduces to the usual expression, $Q(M) = 1/M^2$.

- *Test tone-to-noise ratio referred to an SSB-FDM*

To compare the various modulation methods it is convenient to express system performance in terms of a test tone-to-noise ratio. As discussed previously, this ratio is obtained by subtracting the Holbrook and Dixon loading factor of 9.5 dB from the audio signal-to-noise ratio. Assuming that psophometric weighting of the noise spectrum produces a 3.5 dB improvement, we have

$$\left(\frac{T.T.}{N}\right) = \left(\frac{S}{N}\right)_{dB} - 6. \tag{13}$$

Therefore, from Eqs. (7) and (13)

$$\left(\frac{T.T.}{N}\right) = 2.2\eta^2 - 10 \log_{10}(M-1) - 10 \log_{10}\left[1 + \frac{Q(M)}{M-1} \exp \tfrac{1}{2}\eta^2\right] - 6. \tag{14}$$

To derive an expression for $T.T./N$ in terms of the reference K-channel SSB-FDM system, one must first derive an expression for the average signal power-to-mean square noise ratio at the output of the matched filter, η^2, in terms of the signal parameters at the input to the matched filter receiver. From this, one can derive an expression for η^2 as a function of the equivalent signal-to-noise ratio (P_s/N_k) in a K-channel SSB-FDM system, thereby allowing one to relate $T.T./N$ to the reference system.

We will now derive the signal-to-total-noise ratio at the output of a correlation receiver. For this calculation, it will be assumed that the clutter is a random process having a flat spectrum across the rf bandwidth and that samples of the two Hilbert components measured at points in time separated by the reciprocal of the bandwidth are statistically independent. This assumption will generally be satisfied in all PN signalling techniques where the phases from sample-to-sample are pseudo-randomized intentionally. Hence, binary PN signals (continuous in time) whose period exceeds the total message length will satisfy this condition. The clutter contributed to each correlation measurement even when a single PN interfering signal is active consists of a random collection of sample values and, hence, the assumption is valid even in this case.

Where the same PN signal is used for each message sample, as in digital PPM, the assumptions are still valid since the phase of each signal is randomized by the time modulation from sample to sample. Hence, the calculation for the correlator output signal-to-noise ratio will be sufficiently accurate provided the collection of clutter signals which overlap the desired signal changes randomly from correlation measurement to correlation measurement.

In order to use the error probability expression, Eq. (6), it is essential that the random process at the output of the correlator be white Gaussian. If the bandwidth-time product (WT) per signal is large, as in the case of PN transmission, the correlator output consists of the sum of many independent samples from the same probability distribution function, which causes the output to approach a Gaussian process. This tendency is increased even further when a large number of PN signals are using the satellite simultaneously since here the input process to the correlator is also tending toward a Gaussian process. We therefore expect our results to be quite representative of the true physical situation.

The fraction of the satellite power, P_d, which goes into the desired signal is given by

$$P_d = P(P_u/P_t)L, \tag{15}$$

where

P = the effective received power at the ground station,
P_t = the total power at the satellite receiver
P_u = the up-link power per ground station
L = loss factor, $\frac{1}{4} \leq L < 1$, which accounts for degradation due to hard limiting.

The ratio of P_t to P_u is obtained from

$$P_t/P_u = 1 + d(K-1) + (2WN_u/P_u), \tag{16}$$

where

d = activity factor
K = number of channels
$2WN_u$ = satellite receiver noise power
$2W$ = rf bandwidth; i.e., the signal bandwidth
N_u = satellite receiver noise power density (watts/cps).

The total noise power, σ_t^2, at the ground receiver consists of receiver self-noise plus the component contributed by all the interfering signals in the satellite receiver, i.e., other users and satellite receiver noise. Hence,

$$\sigma_t^2 = 2WN_0 + P[1 - (P_u/P_t)L], \tag{17}$$

where N_0 is the noise power density (watts/cps) of the ground receiver. The signal-to-noise ratio η_{in}^2 at the input to the correlator receiver is

$$\eta_{in}^2 = \frac{P_d}{\sigma_t^2} = \frac{P(P_u/P_t)L}{2WN_0 + P[1 - (P_u/P_t)L]}. \tag{18}$$

The correlator output signal-to-noise ratio η^2 is η_{in}^2 multiplied by the processing gain, $2WT$. Hence, after some manipulation we have

$$\eta^2 = \frac{PTL}{dKN_0} \Bigg/ \left[\frac{P}{2WN_0}\left(1 - \frac{1}{K} + \frac{1-L}{dK} + \frac{2WN_u}{P_{u(\text{avg})}}\right) + \left(1 + \frac{1-d}{dK} + \frac{2WN_u}{P_{u(\text{avg})}}\right)\right]. \tag{19}$$

where

$P_{u(\text{avg})} = dKP_u$ = total (average) ground station transmitter power.

When $N_u = 0$, $d(K+1) \gg 1$, and $L = 1$,

$$\eta^2 = \frac{(PT/dK)}{(P/2W)(1 - 1/K) + N_0}. \tag{20}$$

The assumptions of Eq. (20) are those used throughout the paper.

Equation (19) shows that as the rf bandwidth, W, increases, the mutual signal interference is reduced, and the satellite receiver noise power increases. It is therefore impossible to reduce the clutter completely by increasing rf bandwidth since performance becomes satellite-receiver noise limited.

By differentiating Eq. (19) with respect to W, we can find the rf bandwidth which maximizes the correlator signal-to-noise ratio. Assuming $L = 1$, this bandwidth is given by

$$2W_{\text{opt}} = \left[\frac{P}{N_0}\frac{P_{u(\text{avg})}}{N_u}\left(1 - \frac{1}{K}\right)\right]^{1/2}. \tag{21}$$

Assume that the up and down parameters of the satellite communication link are the same except for the transmitted powers involved. Then, $N_0 = N_u$. Also, let the ground station radiated power be R times the satellite power, i.e., $P_{\text{avg}} = RP$. Then,

$$2W_{\text{opt}} = \frac{P}{N_0}\left[R\left(1 - \frac{1}{K}\right)\right]^{1/2} \cong \frac{P}{N_0}R^{1/2} \quad \text{for} \quad K \gg 1. \tag{22}$$

For large ground stations $R \cong 1000$; then the optimum rf bandwidth is approximately 30 times as large as P/N_0. If $P/N_0 = 10^7$ cps, the rf bandwidth is about 30×10^7 cps. In this case, performance is for all practical purposes thermal noise limited in the down link.

If we substitute Eq. (22) into Eq. (19) (assuming $L = 1$), then

$$\eta_{\text{opt}}^2 = \frac{(PT/dKN_0)}{1 + \frac{1-d}{dK} + 2R^{1/2} + \frac{1}{R}} \cong PT/dKN_0. \tag{23}$$

In most practical situations operating with an optimum bandwidth will yield thermal noise limited performance. In this case, the PN system is equivalent to time-division multiplexing (where dK is the number of active channels), the clutter penalty being negligibly small. Since correlation reception is used with M-ary alphabets, the PN modulation system is also optimum. Now, in Eq. (20) let

$$N_t \equiv \frac{K}{T}\left[\frac{P(1 - 1/K)}{2W} + N_0\right] = \frac{K}{T}N_{t0}, \tag{24}$$

where N_{t0} = clutter energy, N_{c0}, plus thermal noise energy, N_0. Equation (24) represents the situation of a system using a narrow band signal pulse of duration T, power P, and having additive thermal noise power KN_{t0}/T in the equivalent narrow band channel. We can now replace P by P_s, the equivalent full load sinusoid power, since AGC or a hard-limiter precedes the TWT. When the signals received at the satellite have equal power, as we assume here, amplitude nonlinearities have very little effect on the performance of a PN system. Therefore,

$$\eta^2 = P_s/N_t. \tag{25}$$

If the signals have an arbitrary duty (or activity) factor $d \leq 1$, then

$$\eta^2 = P_s/dN_t. \tag{26}$$

To restate this expression in terms of the reference SSB-FDM voice system, we write

$$\eta^2 = \frac{1}{d}\left(\frac{P_s}{N_K}\right)\left(\frac{N_K}{N_t}\right), \tag{27}$$

where

$$N_K = KN_0W_0$$
$$W_0 = 4000 \text{ cps}. \tag{28}$$

Equation (28) is the noise power in a conventional K-channel voice system. Then from Eqs. (24) and (28) we have

$$\frac{N_K}{N_t} = \frac{N_K 2WT}{K[P_s(1 - 1/K) + 2WN_0]}$$
$$= \frac{W_0 T}{\left(\frac{P_s}{N_K}\right)\left(\frac{W_0}{2W}\right)(K-1) + 1}. \tag{29}$$

Combining Eqs. (27), (29), and (11),

$$\eta^2 = \frac{1}{d}\left(\frac{P_s}{N_K}\right)\frac{W_0 T}{\left(\frac{P_s}{N_K}\right)\left(\frac{W_0}{2W}\right)(K-1) + 1}$$
$$= \frac{1}{d}\left(\frac{P_s}{N_K}\right)\frac{1}{\left[\left(\frac{P_s}{N_K}\right)\left(\frac{W_0}{2W}\right)(K-1) + 1\right]2q}. \tag{30}$$

Substituting Eq. (30) into Eq. (14), we arrive at the expression for $(T.T./N)$ of a digital PN system as a function of the communication channel parameters of an SSB-FDM system.

$$\left(\frac{T.T.}{N}\right) = 2.2\frac{1}{d}\left(\frac{P_s}{N_K}\right)\frac{1}{\left[\left(\frac{P_s}{N_K}\right)\left(\frac{W_0}{2W}\right)(K-1) + 1\right]2q}$$
$$- 10\log_{10}(M-1) - 10\log_{10}\left\{1 + \frac{M^{-2q}}{M-1}\right.$$
$$\left.\cdot \exp\left[\frac{1}{2d}\left(\frac{P_s}{N_K}\right)\frac{1}{\left[\left(\frac{P_s}{N_K}\right)\left(\frac{W_0}{2W}\right)(K-1)+1\right]2q}\right]\right\} - 6. \tag{31}$$

As a check of this equation against the Stewart and Huber[1] result, allow the rf bandwidth to approach infinity, thus giving the ideal performance that would be obtained if the signals had zero mutual interference, i.e., the "thermal noise limited" case. Then,

$$\left(\frac{T.T.}{N}\right)_\infty = 2.2\frac{1}{d}\left(\frac{P_s}{N_K}\right)\frac{1}{2q} - 10\log_{10}(M-1)$$
$$- 10\log_{10}\left\{1 + \frac{M^{-2q}}{M-1}\exp\left[\frac{1}{2d}\left(\frac{P_s}{N_K}\right)\frac{1}{2q}\right]\right\} - 6. \tag{32}$$

Assuming that quantization noise is small relative to decision noise

$$\left(\frac{T.T.}{N}\right)_\infty = 2.2\frac{1}{d}\left(\frac{P_s}{N_K}\right)\frac{1}{2q}$$
$$- 10\log_{10}(M-1) - 6. \tag{33}$$

So, for on-off binary transmission using bit-by-bit decision (i.e., $m = 1$ and $M = 2$) where $d = 1/2$ and q is the number of bits per sample,

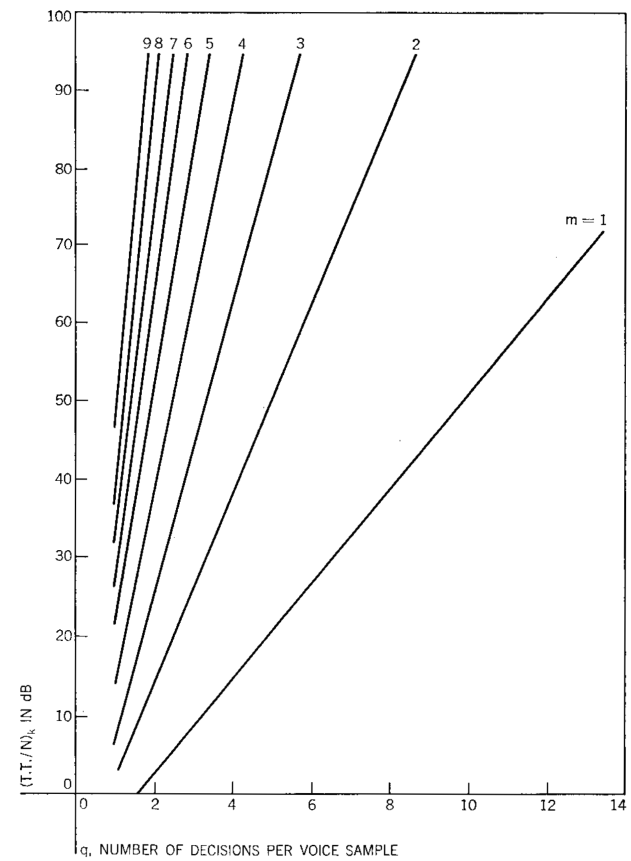
$$\left(\frac{T.T.}{N}\right)_\infty = 2.2\left(\frac{P_s}{N_K}\right)\frac{1}{q} - 6, \tag{34}$$

which is the expression given by Stewart and Huber.[1] For the M-ary alphabet with $M = 2^m \gg 1$ and $q = 1$ (one decision per sample),

$$\left(\frac{T.T.}{N}\right)_\infty = \frac{1.1}{d}\left(\frac{P_s}{N_K}\right) - 3m - 6, \tag{35}$$

where m is the number of bits per decision.

Figure 3 Test tone-to-noise ratio $(T.T./N)_k$ for equal quantization and decision noise. The parameter m is the number of bits per decision.



- *Interpretation of analysis*

An equation has been derived which puts the performance of a digital pseudo-noise system into a "standard" form. With Eq. (31), performance of the system can be compared with conventional modulation techniques. This equation is general in the sense that it is applicable to systems using conventional binary signals, as well as to those using M-ary signal alphabets.

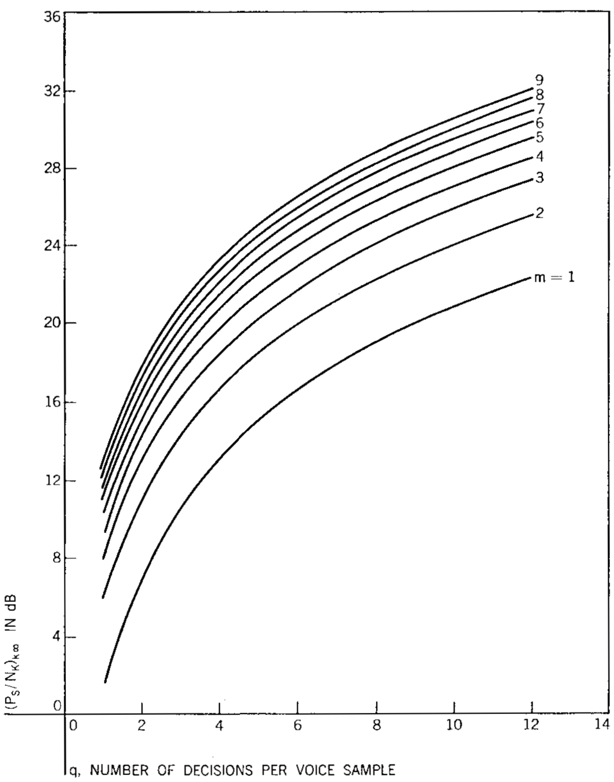
As a further aid to making comparisons, we now examine the operation when the quantization and thermal noise are assumed to be equal. This condition is known as operation at the "knee" of the $T.T./N$ vs P_s/N_K curve. It can be represented by the expression

$$\frac{M^{-2q}}{M-1} \exp \left\{ \frac{1}{2d} \left(\frac{P_s}{N_K} \right) \cdot \frac{1}{\left[\left(\frac{P_s}{N_K} \right) \left(\frac{W_0}{2W} \right) (K-1) + 1 \right] 2q} \right\} = 1. \quad (36)$$

Substituting Eq. (36) into Eq. (31) gives the value of $T.T./N$ for operation at the knee:

$$(T.T./N)_k = 3(2mq - 3). \quad (37)$$

Figure 4 Thermal-noise limited intrinsic signal-to-noise ratio $(P_s/N_K)_{k\infty}$ for equal quantization and decision noise.



Equation (37) is graphed in Fig. 3 as a function of q with m as a parameter.

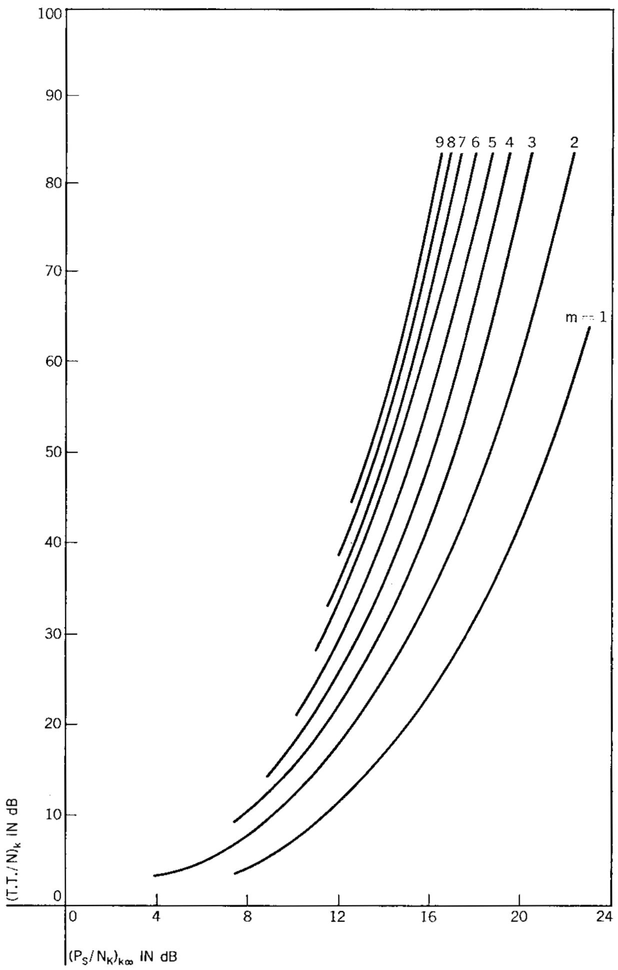
Now, if Eq. (36) is solved for P_s/N_K, the intrinsic signal-to-noise ratio of the reference system is obtained for operation at the knee:

$$\left(\frac{P_s}{N_K} \right)_k = \frac{4qd[2q \ln M + \ln (M-1)]}{1 - \frac{W_0}{2W}(K-1)[2q \ln M + \ln (M-1)]4qd}. \quad (38)$$

Allowing $2W \rightarrow \infty$, the thermal noise limited value is obtained:

$$\left(\frac{P_s}{N_K} \right)_{k\infty} = 4qd[2q \ln M + \ln (M-1)]. \quad (39)$$

Figure 5 $(T.T./N)_k$ vs $(P_s/N_K)_{k\infty}$. Points on these curves are obtained by using the same values of m and q on Figs. 3 and 4.



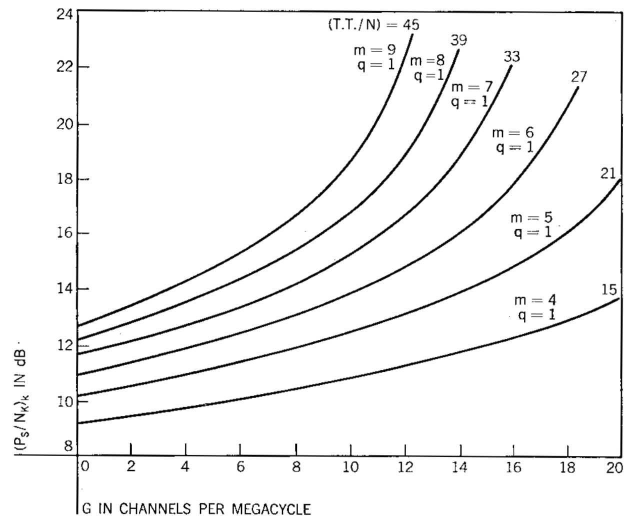
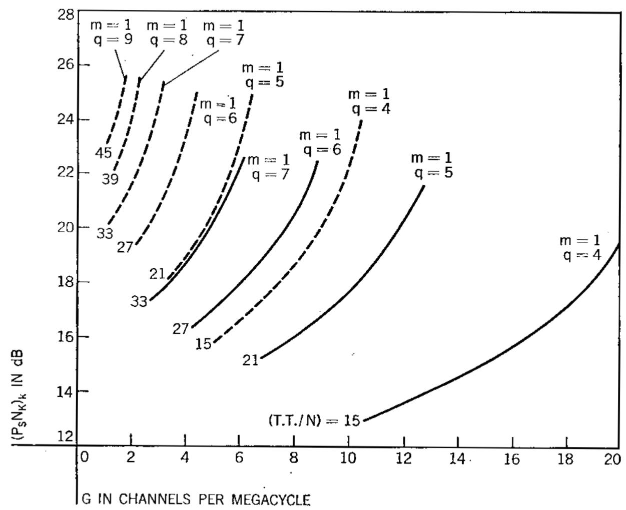


Figure 6 Intrinsic signal-to-noise ratio vs number of channels per megacycle for PN system using M-ary signal alphabet.

Figure 7 Intrinsic signal-to-noise ratio vs number of channels per megacycle for on-off binary signals (solid line) and orthogonal binary signals (dashed line).



$10 \log_{10}(P_s/N_K)_{k\infty}$ is graphed in Fig. 4 as a function of q with m as a parameter and $d = 1/4$. The relationship between $(T.T./N)_k$ and $(P_s/N_K)_{k\infty}$ is obtained from Figs. 3 and 4 and is plotted in Fig. 5. For a given value of m, the pair of values for $\{(T.T./N)_k, (P_s/N_K)_{k\infty}\}$ is determined at the same value of q. The curves in Fig. 5 are thus the envelopes of the $T.T./N$ vs P_s/N_K "knees" at the threshold. It can be seen from Fig. 5 that to obtain, for example, a 45dB $(T.T./N)_k$ with a nine-bit M-ary signal alphabet ($m = 9, q = 1$), an intrinsic signal-to-noise ratio $(P_s/N_K)_{k\infty}$ of 12.8 dB is required. Using a three word per sample alphabet ($m = 3$, $q = 3$) requires $(P_s/N_K)_{k\infty}$ of 16.4 dB for the same quality. On the other hand, a bit-by-bit orthogonal decision procedure ($m = 1, q = 9$) requires $(P_s/N_K)_{k\infty}$ of 20.6 dB. Thus, for a given quality, the M-ary signal alphabet gives the most efficient performance from the standpoint of intrinsic signal-to-noise ratio.

We now develop a computational procedure for calculating $(P_s/N_K)_k$. To do this, we narrow the rf bandwidth to increase the clutter power density, and at the same time increase $(P_s/N_K)_k$ in such a way that $(T.T./N)_k$ stays constant. If this process is continued to the limit, the channel becomes clutter limited at the point where clutter channel capacity equals the thermal noise channel capacity that is required to maintain a constant $(T.T./N)_k$.

It is clear from Eqs. (38) and (39) that

$$\left(\frac{P_s}{N_K}\right)_k = \frac{(P_s/N_K)_{k\infty}}{1 - \dfrac{W_0}{2W}(K - 1)(P_s/N_K)_{k\infty}}. \tag{40}$$

Now, we define a penalty function, Q, which must be applied to the intrinsic signal to noise ratio to take account of the fact that the rf bandwidth is finite:

$$Q = 10 \log_{10} (P_s/N_K)_k - 10 \log_{10} (P_s/N_K)_{k\infty}$$

$$= 10 \log_{10} \left[\frac{\dfrac{2W}{W_0(K-1)(P_s/N_K)_{k\infty}}}{\dfrac{2W}{W_0(K-1)(P_s/N_K)_{k\infty}} - 1} \right]. \tag{41}$$

Let $K/2W = G$, the number of channels per magacycle, and let the audio bandwidth $W_0 = 4{,}000$ cps. Assuming $K \gg 1$, Eq. (41) can be written

$$Q = 10 \log_{10} \left[\frac{250}{250 - G(P_s/N_K)_{k\infty}} \right]. \tag{42}$$

The value of $(P_s/N_K)_{k\infty}$ for a desired $(T.T./N)_k$ can be obtained from Fig. 5. Inserting this $(P_s/N_K)_{k\infty}$ into Eq. (42) allows computation of the penalty Q as a function of G. Then, by the expression

$$10 \log_{10} (P_s/N_K)_k = 10 \log_{10} (P_s/N_K)_{k\infty} + Q, \tag{43}$$

we are able to plot $(P_s/N_K)_{k\infty}$ as a function of G. This is done for several signal alphabets in Figs. 6 and 7.

From Eq. (42), the clutter limited case occurs when $Q = \infty$. For this condition, the clutter limited channel capacity is given by

$$\frac{1}{G_c} = W_0 \left(\frac{P_s}{N_K}\right)_{k\infty} \times 10^{-6} \text{ Mc/sec/channel} \tag{44}$$

Since the thermal noise limited channel capacity is given by

$$\frac{1}{G_n} = \frac{1}{K}\left(\frac{P_s}{N_0}\right) \times 10^{-6} \text{ Mc/sec/channel} \tag{45}$$

and since

$$\left(\frac{P_s}{N_K}\right)_{k\infty} = \frac{P_s}{N_0 K W_0}, \tag{46}$$

we have

$$G_n = G_c. \tag{47}$$

Thus, to maintain a constant $(T.T./N)_k$ it is necessary to maintain a constant channel capacity.

It is clear from the curves shown in Figs. 6 and 7 that there are many acceptable operating points for a given voice quality. Where power and bandwidth are at a premium, a convenient operating point may be chosen by defining a penalty function

$$U = \frac{1}{G} 10 \log_{10} (P_s/N_K)_k \tag{48}$$

for a given $(T.T./N)_k$. Thus, U is directly proportional to the product of power and the bandwidth per channel. We can therefore graph U vs 10 $\log_{10}(P_s N_K)_k$ as shown in Fig. 8 and operate at the point where U is minimum. This penalty function exchanges bandwidth for $(P_s N_K)_k$ in dB.

Where downlink power is at a premium and rf bandwidth is not, this penalty function is not applicable. In this case, the system would be designed for the optimum bandwidth given in Eq. (21). At this operating point, the performance is essentially thermal-noise limited.

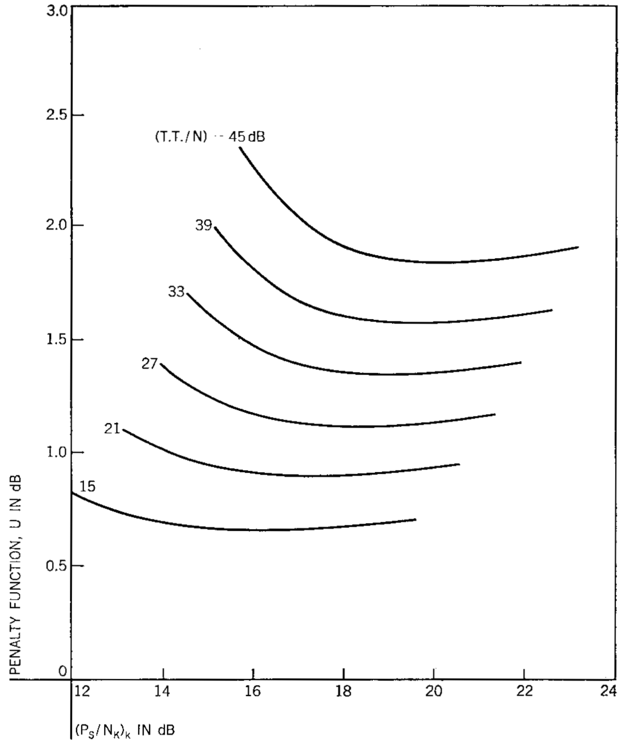


Figure 8 Multiplexing penalty function U vs intrinsic signal-to-noise ratio for M-ary PN signals.

Analysis of analog PN multiplexing

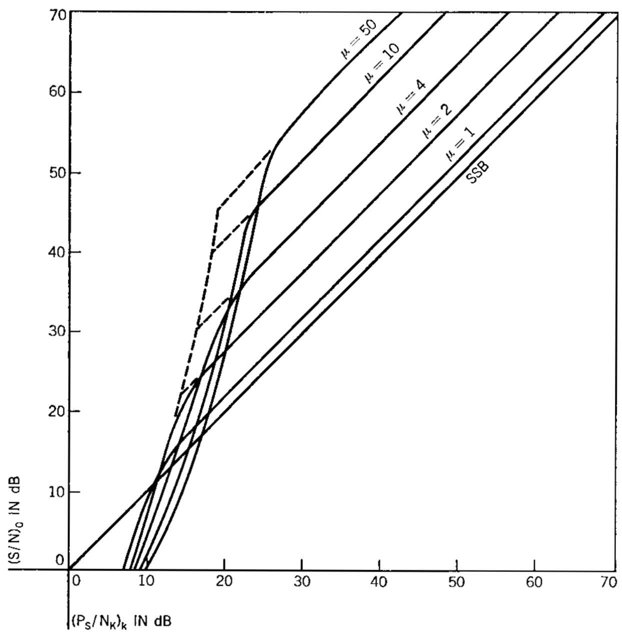
In order to develop the theory of PN-multiplexing for voice signals which frequency modulate a sinusoidal subcarrier, it is convenient to use a mathematical model devised by Akima.[9] In this model the audio output signal-to-noise ratio can be calculated along with the threshold characteristic which approximates that which would be obtained with feedback. An interesting property of this model is that it postulates an M-ary decision procedure for locating the filter that contains the desired signal, which is then converted to an analog voltage by an FM discriminator. The threshold characteristic of this model, which approximates that of frequency modulation with feedback is strongly influenced by the M-ary decision error probability, which was used for the study of digital techniques. Thus, the theory previously developed closely resembles the FM model used here. In Akima's model[9] the audio-output signal-to-noise ratio is given by

$$\left(\frac{S}{N}\right)_0 = \frac{3}{2}\mu^2\eta_p^2 \frac{[1-(\mu+1)\alpha_0]^2}{1+\mu(\mu+1)(\mu+2)\alpha_0\eta_p^2}, \tag{49}$$

where

$$\eta_p^2 = 2\eta^2 = \frac{1}{d}\left(\frac{P_s}{N_K}\right)\frac{1}{\left(\frac{P_s}{N_K}\right)\left(\frac{W_0}{2W}\right)(K-1)+1} \tag{50}$$

Figure 9 Audio signal-to-noise ratio vs intrinsic signal-to-noise ratio for FM (from Akima[9]). Dashed lines show effect of feedback.



and μ is the FM index. Equation (49) can be plotted as a function of (P_s/N_k) and these curves are presented for reference in Fig. 9. The knee of the threshold characteristic is obtained by equating the competing noises just as in the digital case. Hence,

$$\alpha_0 = \frac{1}{2\eta^2\mu(\mu+1)(\mu+2)}, \tag{51}$$

where

$$\alpha_0 = \frac{\alpha}{\mu} = \tfrac{1}{2}\exp\left(-\tfrac{1}{2}\eta^2\right). \tag{52}$$

(Here, the FM index μ is equal to $(M+1)$ where M represents the number of filters used in the model.) At this point $(S/N)_0$ is for all practical purposes

$$\left(\frac{S}{N}\right)_0 = \tfrac{3}{2}\mu^2\eta^2. \tag{53}$$

At the threshold,

$$\exp\left(-\tfrac{1}{2}\eta^2\right) = \eta^2\mu(\mu+1)(\mu+2). \tag{54}$$

Let η_μ^2 be the input signal-to-noise ratio at the threshold of the FMFB receiver, obtained from Eq. (54). From Eq. (50)

$$\left(\frac{P_s}{N_K}\right)_k = \frac{2d\eta_\mu^2}{1-\frac{W_0}{2W}(K-1)2d\eta_\mu^2}. \tag{55}$$

When $W \to \infty$, we obtain the threshold signal-to-noise ratio when performance is thermal noise limited only. Then, from Eq. (55)

$$\left(\frac{P_s}{N_K}\right)_{k\infty} = 2d\eta_\mu^2. \tag{56}$$

As in the case of digital transmission, we have here,

$$\left(\frac{P_s}{N_K}\right)_k = \frac{(P_s/N_K)_{k\infty}}{1-\frac{W_0}{2W}(K-1)(P_s/N_K)_{k\infty}}. \tag{57}$$

The computation procedure for $(P_s/N_K)_k$ versus G is the same as before and results in the performance curves shown in Fig. 10. Curves showing the penalty function, U, are also derived as before, and are shown in Fig. 11 for the PN-FMFB system.

Above threshold, the decision error noise can be neglected since performance is limited by thermal noise and clutter. Then,

$$\left(\frac{T.T.}{N}\right) = 10\log_{10}\frac{1}{d}\left(\frac{P_s}{N_K}\right) - 10\log_{10}\left[\left(\frac{P_s}{N_K}\right)\left(\frac{W_0}{2W}\right)(K-1)+1\right] + 20\log_{10}\mu - 4. \tag{58}$$

When $W \to \infty$,

$$\left(\frac{T.T.}{N}\right)_\infty = 10\log_{10}\frac{1}{d}\left(\frac{P_s}{N_K}\right) + 20\log_{10}\mu - 4. \tag{59}$$

Once again, when the PN-multiplexing loss is neglected by letting $W \to \infty$, $(T.T./N)$ takes on the same form as for conventional FM above threshold.

The additional constraint is that for a given FM modulation index the intrinsic signal-to-noise ratio must be above the threshold so that Eq. (59) holds. This can easily be checked by using Eq. (55).

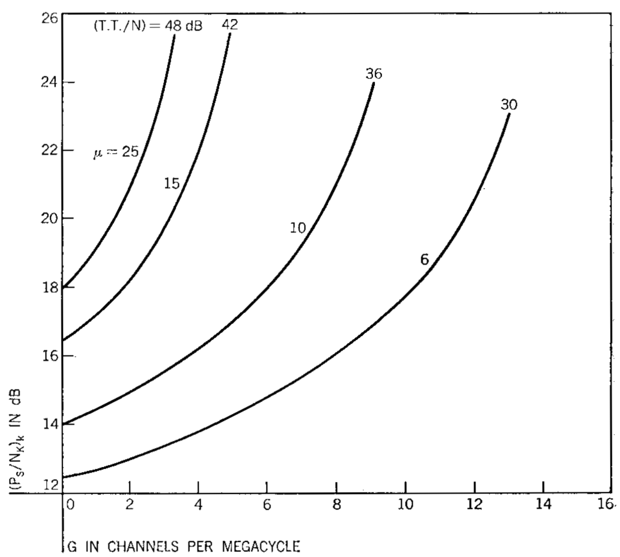
Comparison of modulation techniques

In this section we will compare the modulation techniques for a given test tone-to-noise ratio. In the conventional analog systems, we will assume that the test tone-to-thermal noise ratio is equal to the test tone-to-distortion ratio. Thus, if we compute the test tone-to-thermal noise ratio, we will subtract 3 dB to obtain the test tone-to-total noise ratio. In PN-multiplexing, the computations automatically include the total distortion.

We will compare the performance of digital PN systems with that of analog PN systems, the performance of various conventional systems with each other, and finally, the performance of PN with respect to conventional systems. Comparison will be based on the values of the intrinsic signal-to-noise ratio and the number of channels per megacycle of rf bandwidth. The values of these parameters will be determined for a given quality of performance specified by the test tone-to-total noise ratio. The choice of modulation parameters will reflect practical considerations.

For those techniques using digital transmission, we will operate above the knee of the operating characteristic, by

Figure 10 Intrinsic signal-to-noise ratio vs number of channels per megacycle for PN-FMFB system.



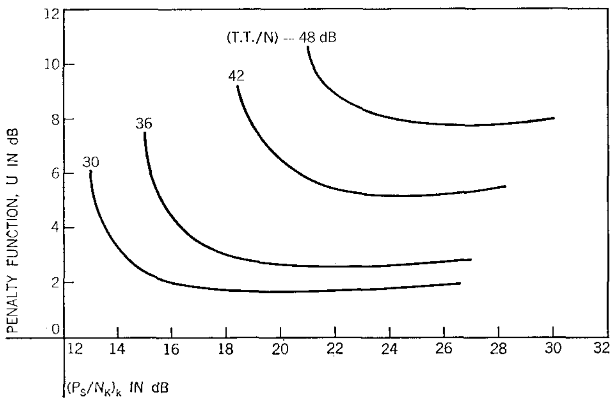


Figure 11 Multiplexing penalty function U vs intrinsic signal-to-noise ratio for PN-FMFB system.

increasing (P_s/N_K) by 3 dB. We will also increase the value of $T.T./N$ by 3 dB so that the acceptable range of $T.T./N$ will actually be between 45 and 48 dB.

We will assume a 25 percent activity factor in all PN systems, recognizing that in PN-FMFB, on-off operation is more difficult to achieve. The results obtained assume that rf phase information is not used, although in the coherent techniques, this type of operation will improve performance. Our results will therefore be conservative in this case. However, for good voice quality, performance is only slightly improved when rf phase-lock is used, except in the special case of biphase modulation. Here, we add a 3 dB improvement in the intrinsic signal-to-noise ratio and double the channels per megacycle.

All systems will be classified into three categories:

(1) Very good quality, $T.T./N = 48$ dB
(2) Good quality, $T.T./N = 42$ dB
(3) Acceptable quality, $T.T./N = 36$ dB

We will require at least two channels per megacycle. Any modulation technique which cannot satisfy this will be automatically eliminated. Thus, a 200-channel system will require a satellite bandwidth less than 100 Mc/sec.

- *Comparison of PN-multiplexing techniques*

The comparison here is shown in Table 1. The two best PN modulation techniques are: M-ary digital message transmission and PN-FMFB. The choice of the optimum modulation technique is based on the minimum value of the penalty function, U. It is quite clear that the M-ary system is far more efficient than PN-FMFB for the indicated parameters. For acceptable voice quality ($T.T./N =$ 36 dB), PN-FMFB is competitive, particularly where power and not bandwidth is at a premium, as is the case in satellite communications.

Table 2 shows the parameter values of the M-ary system when the penalty function is displaced from the minimum to reduce satellite power. This tradeoff causes only a slight decrease in the number of channels per megacycle. The penalty function is also increased only slightly. Based on the penalty function, performance is suboptimum, although still substantially better than PN-FMFB. These parameters are perhaps more useful than those based exactly on the minimum value of the penalty function, since less satellite power is required here with only a small loss in the channels per megacycle. These results will be compared with FMFB using conventional multiplexing.

- *Comparison of conventional multiplexing techniques*[1]

Table 3 shows a comparison of three conventional multiplexing techniques based on the intrinsic signal-to-noise ratio and the number of channels per megacycle. Clearly, SSB has the maximum number of channels per megacycle of any modulation system, but it also requires substantially more power than composite FMFB and on-off conventional PCM. However, composite FMFB is superior to PCM in satellite power requirements. In the region of medium-to-acceptable voice quality, FMFB also obtains more channels per megacycle than PCM. Where bandwidth is at a premium and the power constraint is somewhat relaxed, SSB and narrow deviation FM ($\mu = 1$) are reasonable modulation techniques for acceptable voice quality. For high quality systems, it is necessary to exchange channels per megacycle for on-board power.

- *PN-multiplexing compared to conventional*

Table 4 shows the comparison between the PN M-ary system, and conventional multiplexing using composite FMFB. The table shows that for very good quality, PN-

Table 1 Comparison of digital and analog PN Techniques.

Type of System	$(T.T./N)$ in dB	(P_s/N_K) in dB	G in channels per Mc/sec	U in dB
M-ary PN: $m = 9, q = 1$	48	23.0	10.8	2.13
PN-FMFB: $\mu = 25$	48	27.0	3.46	7.80
M-ary PN: $m = 8, q = 1$	42	22.5	12.3	1.83
PN-FMFB: $\mu = 15$	42	24.5	4.8	5.10
M-ary PN: $m = 7, q = 1$	36	22.0	14.0	1.57
PN-FMFB: $\mu = 10$	36	21.0	8.1	2.60

Table 2 Parameters for M-ary PN system where U is not minimum.

Type of System	$(T.T./N)$ in dB	(P_s/N_K) in dB	G in channels per Mc/sec	U in dB
M-ary PN: $m = 9, q = 1$	48	21	9.4	2.24
M-ary PN: $m = 8, q = 1$	42	21	11.2	1.88
M-ary PN: $m = 7, q = 1$	36	20	12.2	1.64

Table 3 Comparison of conventional modulation techniques (derived from Stewart and Huber[1]).

Type of System	$(T.T./N)$ in dB	(P_s/N_K) in dB	G in channels per Mc/sec
SSB	48	42	250
Composite FMFB: $\mu = 10$	48	20	11.4
On-off PCM: $q = 9$	48	26.6	13.9
SSB	42	36	250
Composite FMFB: $\mu = 4$	42	22	25
On-off PCM: $q = 8$	42	25.6	15.6
SSB	36	30	250
Composite FMFB: $\mu = 2$	36	22	41.6
On-off PCM: $q = 7$	36	24.4	17.8

multiplexing and large index FMFB ($\mu = 10$) are comparable as far as the channel parameters are concerned. However, for good and acceptable quality, FMFB using relatively narrow deviation FM makes better use of the channel bandwidth. It is therefore quite clear that PN-multiplexing is competitive with FMFB where toll quality performance is required. The penalty factor which is useful for making a choice among PN systems is not very useful for comparing PN systems to conventional ones, since the latter are inherently far more efficient in bandwidth for average quality. The penalty function favors systems which make efficient use of bandwidth, since it exchanges bandwidth for power.

The numbers which have been chosen in this section for the comparison of conventional and PN-multiplexing techniques indicate that approximately the same values of P_s/N_K and G can be achieved for both FMFB and PN-multiplexing in the high quality case. Where rf bandwidth is more important than the down-link P_s/N_0, these parameter values represent a good compromise. Here, the number of channels per megacycle, G, is an important comparison criterion. However, where the rf bandwidth is not of primary importance, the number of channels per mega-

Table 4 Comparison of optimum PN and conventional techniques.

Type of System	$(T.T./N)$ in dB	(P_s/N_K) in dB	G in channels per Mc/sec	U in dB
M-ary PN: $m = 9, q = 1$	48	21	9.4	2.24
Composite FMFB: $\mu = 10$	48	20	11.4	1.75
M-ary PN: $m = 8, q = 1$	42	21	11.2	1.88
Composite FMFB: $\mu = 4$	42	22	25.0	0.88
M-ary PN: $m = 7; q = 1$	36	20	12.2	1.64
Composite FMFB: $\mu = 2$	36	22	41.6	0.53

Table 5 Comparison of Composite FMFB and M-ary PN with infinite rf bandwidth.

Type of System	$(T.T./N)$ in dB	(P_s/N_K) in dB	G in channels per Mc/sec	U in dB
M-ary PN: $m = 9, q = 1$	48	15.7	0	∞
Composite FMFB: $\mu = 10$	48	20	11.4	1.75
M-ary PN: $m = 8, q = 1$	42	15.2	0	∞
Composite FMFB: $\mu = 4$	42	22	25.0	0.88
M-ary PN: $m = 7, q = 1$	36	14.7	0	∞
Composite FMFB: $\mu = 2$	36	22	41.6	0.53

cycle is not significant as a comparison criterion. The quantity P_s/N_K is much more significant since it then becomes the limiting factor on system performance. Under these conditions, PN-multiplexing using M-ary alphabets is substantially superior to FMFB as shown in Table 5. Whereas conventional FMFB requires an rf bandwidth which is several times less than P_s/N_0, PN modulation operates most efficiently when the bandwidth is much greater than P_s/N_0.

Summary and conclusions

Pseudo-noise modulation theory has been put into the "standard form" which allows it to be compared with more conventional radio telephone systems. Although the empirical load factors may be different for different telephone systems, the theoretical results remain the same and can easily be modified to obtain the test-tone-to-total-noise ratio.

The results of this study demonstrate that the *M*-ary alphabet is essential in order to make PN-systems as efficient as conventional wide-band systems such as FMFB, for high quality telephony. Above $T.T./N = 48$ dB, pseudo-noise *M*-ary systems which take advantage of the voice channel activity factor can be as efficient as wide-band FMFB both in power and bandwidth utilization. Since on-board power is at a premium, the useful modulation systems must exchange power for bandwidth to achieve a high quality voice channel. For reduced voice quality, i.e., $T.T./N = 36$ dB, narrow deviation FMFB is substantially more efficient than PN as far as rf bandwidth is concerned.

Where power is at a premium and bandwidth is not, a pseudo-noise satellite system is equivalent to an orthogonal multiplexing system having a negligible multiplexing penalty. In addition, since the repeated signal is spread over an extremely wide band, the power density radiated at the ground is small, minimizing interference with existing ground-based microwave links.

Our results show that PN-FMFB with practical deviations and PN-PCM (i.e., bit-by-bit decisions) are less efficient than pseudo-noise *M*-ary ($M \gg 1$) both in bandwidth and power for high quality voice systems.

From a conventional modulation standpoint, the optimum system chosen was SSB-up, composite FMFB-down. This system requires linear up-link, and because of the complex signal, necessarily uses the peak-limited ground station transmitter inefficiently. In addition, the signal received by the satellite must be converted to frequency modulation which requires more on-board circuit equipment.

The PN-system, on the other hand, is for all practical purposes immune to amplitude nonlinearities from transmission to reception. In addition, the on-board electronics takes on its simplest form.

Power control is required in all the modulation systems except in PCM-TDM, where precise time synchronization of all the ground stations in the net is required.

The major conclusion drawn from this study based on theoretical analysis shows that pseudo-noise is an efficient modulation technique for multiple access satellite communications. The economics of this modulation is intimately tied to reliability and equipment implementation cost and complexity, and to overall system organization and operational requirements. These considerations were beyond the scope of this study. In order to complete this work, it is essential to study the economics of a pseudo-noise communication satellite system and to specify the conditions, system configurations and applications where it would be uneconomical.

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