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Thermal Limitations on the Energy of a Single Injection Laser Light Pulse*

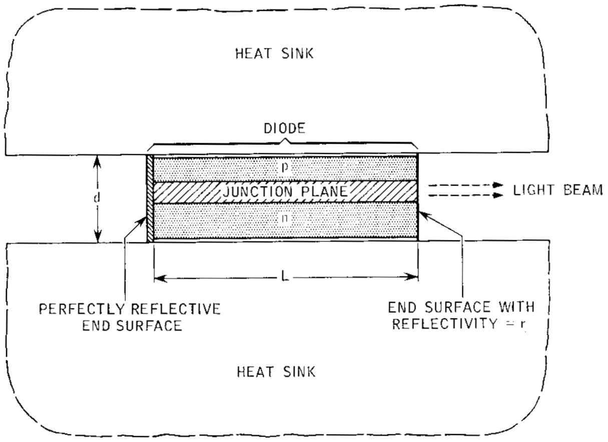
Abstract: The upper limits on the output pulse power of an injection laser arising from heating effects are estimated. The heat is assumed to be dissipated by conduction through a large homogeneous body. A simple method for computing over-all diode efficiency is given.

Introduction

The maximum power that can be generated by an injection laser in the form of coherent light is usually determined by the amount of heat that can be dissipated during operation. The heat is continuously generated in the diode and results in a rise of temperature which lowers the efficiency of generation of light and raises the threshold current density. The object of this paper is to derive upper limits for the emitted coherent optical power assuming certain theoretical and empirical relations that have been shown to apply to GaAs injection lasers. It should be observed that the results probably represent the extreme upper limits on the attainable power generation, because of the possibility that the derivation may neglect some heat sources or some necessary deviations from the ideal geometry of our model.

Figure 1 illustrates the type of device we consider, in which a Fabry-Perot type injection laser is sandwiched between two large heat sinks. The minimum separation between the outer surfaces of the diode and the outer surfaces of the heat sinks must be greater than the heat diffusion distance s which is defined below in Eq. (1).[1] The diode itself has thickness d, width W between the rough sides, and length L between reflecting ends, one of

Figure 1 **An injection laser with ideal heat sinks.** *The reflecting ends of the laser are shown in profile. The end at the left is perfectly reflecting to send all the coherent light into a beam to the right. A view from either end of the diode would be similar except that the dimension W would replace L, and the roughened sides of the diode crystal would appear in profile. The heat sinks are large enough for the temperature rise at their outer surfaces to be negligible.*



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which is perfectly reflecting and the other has reflectivity r. For short pulses the heat diffusion length s will be much less than L or W, and the heat will flow through the diode and heat sink away from the junction in one dimension perpendicular to the plane of the junction. For longer pulses s will be much greater than L, W or d, and the heat flow will take place in three dimensions away from the diode. Another way of characterizing the heat flow for different pulse lengths is that for *short* pulses the surfaces of constant temperature may be approximated by planes parallel to the diode junction plane, and for *long* pulses they may be approximated by spherical surfaces in the heat sinks centered at the diode.

In one respect our assumptions tend to give too low a value for the heat that can be dissipated, and therefore tend to put too low a limit on output power. The assumption that all the heat is produced in a layer of negligible thickness tends to overestimate the temperature rise. The ohmic heating, for example, occurs throughout the diode or at the metal contacts to the diode crystal. For pulse lengths of one microsecond or greater, the heat diffusion length is comparable to or greater than the assumed diode thickness of two mils ($d = 50\mu$) and this correction is not serious.

At low current densities the heat power generated is the sum of the threshold power, P_t; of the power corresponding to an internal quantum efficiency smaller than unity, P_η; and of the power of the reabsorbed coherent light, P_α. At higher current densities the power associated with ohmic heating, P_Ω, becomes important. In our numerical work we shall assume that the electrical resistance is due only to the bulk resistivity of the semiconductor material on either side of the junction. Although in present devices the contact resistance between the metallic contacts and the laser crystal may be more important than the resistance due to bulk resistivity, we have not included the former because of a lack of definite data. This is one important reason why it may be difficult to achieve in practice the power levels computed here.

For one-dimensional heat flow the heat penetrates into the material surrounding the junction a characteristic distance equal to the heat diffusion length,[1] which is given by

$$s = \sqrt{\frac{\pi\kappa\tau}{\mu c_p}}, \tag{1}$$

where κ is the heat conductivity, c_p is the specific heat, μ is the mass density, and τ is the duration of the pulse. For GaAs (See Ref. 2) at 80°K, $\kappa \approx 3$ watts cm^{-1} deg^{-1}, and $\mu c_p \approx 0.85$ joules cm^{-3} deg^{-1}, so that

$$s_{\mathrm{GaAs}}(\mathrm{cm}) \approx 3.3\tau^{1/2}(\mathrm{sec}). \tag{2}$$

By setting the length s of Eq. (2) equal to L or W we can solve for a critical pulse time τ_c. The heat generated by pulses of duration shorter than this will be dissipated by one-dimensional heat flow and that generated by longer pulses will be dissipated by three-dimensional heat flow. We note that τ_c varies quadratically with s so that our subsequent analysis of a range of diode sizes from 10^{-2} cm to 1 cm corresponds to a range in critical pulse lengths of from $\tau_c \approx 10^{-5}$ sec for 10^{-2}-cm diodes to $\tau_c \approx 10^{-1}$ sec for 1-cm diodes. For pulses short compared to τ_c, we use one-dimensional solutions of the transient heat flow problem, in which the heat rise at the junction is proportional to $\tau^{1/2}$. For pulse lengths long compared to τ_c, the temperature rise at the junction levels off to a time-independent value that is characteristic of the solution of the steady state heat flow problem. A number of cases of practical interest fall near the dividing line between the transient and steady state conditions, and hence the proper approximation depends critically upon the diode size considered.

The dissipation of heat by conduction

If the diffusion of heat takes place in one spatial dimension, the heat power will be a function of the allowed temperature rise ΔT at the end of a pulse of time duration τ, or

$$P_Q(\tau) = \sqrt{\pi\kappa\mu c_p}\left(\frac{\Delta T}{\sqrt{\tau}}\right)L W. \tag{3}$$

Equation (3) is derived from the one-dimensional diffusion equation, employing the assumption that the heat is conducted away from both sides of the junction. The temperature change ΔT must be so small that the relevant parameters do not vary greatly during the pulse; the thermal conductivity κ, for example, decreases by a factor of roughly five from 80° to 300°K and we must choose ΔT much smaller than this difference. However, if the junction is somehow imbedded in an effectively infinite medium and the pulse length τ is sufficiently long compared to the critical time τ_c, the heat can diffuse in all three spatial directions. In this situation the temperature at the junction will cease to rise as a function of time and the heat power dissipated becomes independent of pulse duration. Thus

$$P_Q(\infty) \approx 2\sqrt{\pi}\,\kappa\sqrt{L W}\,\Delta T. \tag{4}$$

Equation (4) is derived from the solution of the steady state equation for a heat source $P_Q(\infty)$ distributed over the area $(L \times W)$ of the junction, but makes use of a fictitious geometry in which $(L \times W)$ is replaced by a spherical surface of the same area. In this geometry there is a time-independent solution which is valid for a given radius and a sufficiently long time. This time-independent solution is simply that the temperature rise is proportional to the reciprocal of the distance from the center of the source.

For a device which is on one surface of an effectively semi-infinite heat sink the right-hand element of Eq. (4) should be divided by $\sqrt{2}$.

The heat sources

We shall derive expressions for the four sources of heat: that due to reabsorption of the coherent light by free carrier absorption P_α, that arising from an internal quantum efficiency which is less than unity P_η, that generated by the threshold power P_t, and that due to ohmic heating P_Ω.

The ratio of coherent power output P_0 to total coherent optical power emitted internally is

$$\frac{P_0}{P_\alpha + P_0} = \frac{g - \alpha}{g}, \tag{5}$$

where α is the free carrier absorption coefficient and g is the total gain per unit length due to stimulated emission. The net gain at or above threshold is determined by

$$re^{2(g-\alpha)L} = 1, \tag{6}$$

where L is the length of the diode between reflecting faces, one of which we take to be perfectly reflecting and the other to have reflectivity r. We can now express the reabsorbed optical power as

$$P_\alpha = \frac{LP_0}{L_\alpha}; \qquad L_\alpha = \frac{\ln r^{-1}}{2\alpha}. \tag{7}$$

This result shows that diodes longer than the characteristic length L_α will be quite inefficient because of reabsorption of radiation.[3] We see also that this characteristic length depends strongly on α and less strongly on the reflectivity.

The heat power associated with an internal quantum efficiency η is

$$P_\eta = \frac{1-\eta}{\eta}(P_0 + P_\alpha). \tag{8}$$

We assume, of course, that this energy does not escape the crystal in the form of radiation.

The sub-lasing light emitting efficiency is quite small, and hence we know that almost all of the threshold power P_t contributes to the heating of the device. Careful experiments by M. Pilkuhn and H. Rupprecht[4] have shown that the variation of threshold current with length is consistent with the threshold relation of Eq. (6) if the gain g is assumed to be equal to a constant β times the current density j_t. We can now obtain the dependence of the threshold power on the diode length L as

$$P_t = V_0 L W j_t = \frac{V_0 L W g}{\beta} = \frac{V_0 L_\alpha W \alpha}{\beta}\left(1 + \frac{L}{L_\alpha}\right), \tag{9}$$

where we have used Eqs. (6) and (7) to express the gain g in terms of α, L, and L_α. For diodes formed by diffusing zinc into n-type GaAs with carrier density $n = 0.9 \times 10^{18}$ cm, Pilkuhn and Rupprecht report $\beta = 0.024$ cm/amp at 77°K.

The power used in the injection of carriers (i.e., the total power minus the ohmic heating) is the sum of P_0, P_α, P_η, and P_t. Equating this to the product of current I times the band gap voltage V_0 gives

$$I = \left(1 + \frac{L}{L_\alpha}\right)\left(\frac{\alpha L_\alpha W}{\beta} + \frac{P_0}{\eta V_0}\right). \tag{10}$$

The ohmic heat power P_Ω is the product of the square of this current and the ohmic resistance, which is

$$R = \frac{d\rho}{LW}, \tag{11}$$

where d is the distance between plane parallel contacts and ρ is the average resistivity.

The determination of maximum output power

The sum of the four heat sources discussed in the preceding section is equal to the heat power P_Q that must be dissipated by conduction. The resulting equation is a quadratic in the output power P_0 because, although P_α and P_η are linear in P_0, the ohmic heating P_Ω (which varies as I^2) includes a term in P_0 squared.

It is clear that both the actual dimension L and the characteristic length L_α are important parameters in any design trying to optimize power output and efficiency, yet on the other hand it is always advantageous to make W as large as possible. In the subsequent analysis we will take $W = L$, noting that this should not be considered an upper bound. With suitable precautions to avoid lasing in the W direction, it should be possible to make W several times greater than L.

We will analyze, then, diodes having three values of length $L(10^{-2}$, 10^{-1}, and 1 cm) and two values of $L_\alpha(5 \times 10^{-2}$, and 10^{-1} cm). These correspond to $r = 0.3$ for one uncoated face of GaAs and to $\alpha = 12$ cm^{-1} and 6 cm^{-1}, respectively. We analyze the specific case of a one millisecond pulse. For $\tau = 10^{-3}$ sec, $s_{GaAs} \approx 10^{-1}$ cm [see Eq. (2)], a value which exceeds the thickness of the diode. We will assume good thermal contact to a heat sink of approximately the same thermal properties as GaAs, which surrounds the diode on all sides. (With even the best heat sink materials the energy output increases by less than a factor of two.) We assume $\eta = 70\%$ and $V_0 = 1.48$ volts. The gain per unit current density we take as $\beta = 0.024$ cm amp^{-1}. The diode thickness is assumed to be 5×10^{-3} cm, or 2 mils between contacts, with a resistivity of $10^{-3}\Omega$ cm. We will assume a temperature of 77°K and therefore a heat conductivity[2] of $\kappa = 3$ watts cm^{-1} deg^{-1} and a heat capacity per unit volume of $\mu c_p = 0.85$ joules cm^{-3} deg^{-1}.

Table 1 **Power balance for GaAs laser 0.01 × 0.01 cm, at 77°K: $\tau = 10^{-3}$ sec, $\tau_c \approx 10^{-5}$ sec, R = $5 \times 10^{-2}\ \Omega$.**

α (cm^{-1})	L/L_α	$P_Q(\infty)$ (watts)	P_0 (watts)	I (amps)	P_Ω (watts)	Efficiency $P_0/(P_0 + P_Q)$
12	0.2	4.3	3.8	4.8	1.0	47%
6	0.1	4.3	4.5	5.0	1.3	51%

Table 2 **Power balance for GaAs laser 0.1 × 0.1 cm, at 77°K: $\tau = 10^{-3}$ sec, $\tau_c \approx 10^{-3}$ sec, R = $5 \times 10^{-4}\ \Omega$.**

α (cm^{-1})	L/L_α	$P_Q(\tau)$ (watts)	$P_Q(\infty)$ (watts)	P_0^a (watts)	I (amps)	P_Ω (watts)	Efficiency $P_0/(P_0+P_Q)$
12	2	36	43	7.5	29	0.42	17%
6	1	36	43	15.3	34	0.58	30%

[a] These data correspond to a value of $P_Q = 36$ watts.

Table 3 **Power balance for GaAs laser 1 × 1 cm, at 77°K: $\tau = 10^{-3}$ sec, $\tau_c \approx 10^{-1}$ sec, R = $5 \times 10^{-6}\ \Omega$.**

α (cm^{-1})	L/L_α	$P_Q(\tau)$ (watts)	P_0 (watts)	I (amps)	P_Ω (watts)	Efficiency $P_0/(P_0+P_Q)$
12	20	3600	97	2470	30	2.6
6	10	3600	218	2540	32	5.7

Allowing a temperature rise of 40°C, we compute P_Q from Eq. (3) or Eq. (4), according to which is appropriate. We solve the quadratic equation in P_0 obtained by setting the sum of P_t, P_α, P_η and P_Ω equal to P_Q. The results are shown in Tables 1, 2, and 3.

For the smallest diode, Table 1, the operating conditions are relatively independent of the choice of L_α and, except for pulses less than 10 microseconds in length, the energy balance is determined by steady state conditions. There is an appreciable but not serious ohmic loss. For microsecond pulses the ohmic loss would be the limiting factor.

In the case of the one-millimeter diode given in Table 2 we see that the limits for pulsed and continuous operation are quite comparable so that our numerical values refer almost equally well to either case. Ohmic losses are negligible, and the efficiency and output power depend markedly on the attainable value of L_α. The proportionality of $P_Q(\tau)$ to $\tau^{-1/2}$ means that the choice of a 10-microsecond pulse in our example would increase the output powers by somewhat more than a factor of 10 since the overall efficiency would then be higher (operating further away from threshold) and ohmic losses would still be small.

The results given in Table 3 for the longest and highest-output diode show that the efficiency increases strongly as the light absorption coefficient α decreases. This would indicate that more lightly doped units with their smaller free carrier absorption would be able to produce higher power. The advantage gained in this way would be offset to some extent by the higher threshold resulting from a smaller value of β. In principle, the output of these devices would increase almost linearly with the width W.

Discussion

We see from the foregoing simple considerations that if the ohmic and threshold powers are negligible, the maximum power output for short pulses goes with the inverse square root of the pulse duration, and the total pulse energy with the square root of the pulse duration. (This simplification allows us to equate Eq. (8) with Eq. (3) or (4) and to avoid consideration of the more accurate quadratic solution for P_0). We also use Eq. (7) to eliminate P_α. For longer pulses in devices allowing spherical or hemispherical heat conduction into a large body, the output power becomes independent of pulse length. Equation (4) also gives the heat power for continuous operation if the heat sinks are large and if ΔT is taken as the temperature at the diode minus the temperature maintained at the outer surface of the heat sinks. The heat power in small diodes and short pulses is generated mostly by ohmic heating, and in large diodes it is generated by the reabsorption of radiation. This conclusion is not obvious from Tables 1, 2, and 3, where the pulse length is kept constant with the result that, essentially, we analyze the steady state solution for small diodes. If, however, we choose $\tau < \tau_c$ for the laser dimensions of Table 1 and analyze the transient solution, our allowed peak currents rise. Since the importance of ohmic heating increases quadratically with the current it soon becomes the major limitation on power dissipation.

A limitation not considered here is one that results from the generation of second harmonic radiation at a rate proportional to the square of the internal coherent intensity.[5-8] The second harmonic radiation is strongly absorbed, producing highly excited carriers whose excess energy is turned into heat by emission of phonons. Empirically this phenomenon could be detected by observing a decrease in differential quantum efficiency for high current pulses of such short duration that the heating effects considered here would not produce a sizable change in temperature. One group of observers of this effect[8] estimates that for their particular diode and under operating conditions of their experiment, 1 percent of the light energy was being internally converted to second harmonic radiation. The heating effects we consider here can in principle be greatly reduced as the technology improves, but the second harmonic limitation is a more fundamental one.

The form of Eq. (7) for the self-absorption of the coherent radiation suggests that for long diodes one could obtain higher efficiency by decreasing the reflectivity r of the transmitting end of the diode through the use of an anti-reflection coating. A detailed examination of this possibility shows, however, that no great improvement can be expected. A large value of $\ln r^{-1} = 2L(g - \alpha)$ implies that the stimulated emission does not take place uniformly throughout the junction plane but, rather, is concentrated at the emitting end of the junction. As far as our analysis is concerned such a situation would simply be equivalent to that of a shorter diode with normal reflectivity.

The maximum output power may be increased by increasing the width W of the junction. If the pulse duration t is less than the critical time τ_c, then the heat flow is perpendicular to the junction and the heat power allowed is proportional to W. In this regime, as far as power input and output are concerned, a diode with width $2W$ is equivalent to two diodes with width W. For longer pulses and spherical heat flow the power output will increase with W but more slowly than the first power of W.

The thickness d of 50 microns used in the numerical example is roughly appropriate to present technology. As better methods of making electrical contacts to thin semiconductor layers are developed this thickness will doubtless decrease and one will then be able to use short diodes to obtain higher output power for short pulses.

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