G. J. Lasher

Threshold Relations and Diffraction Loss for Injection Lasers*

Abstract: Mathematical expressions are derived for the minimum current density necessary to cause stimulated emission in injection lasers. A new type of diffraction loss for a thin light-emitting layer surrounded by light-absorbing material is calculated.

Introduction

This paper† presents some threshold relations relevant to the recent achievement of stimulated emission in gallium arsenide diodes by M. I. Nathan et al.[1-3] These devices differ from previously existing lasers in that they are excited by the flow of an electric current across a *p-n* junction, which causes the emission of light in a thin layer adjacent to the junction. We give formulas for the minimum current necessary to obtain coherent light emission in terms of the dimensions of the resonant structure and measurable physical characteristics of injection light sources. Included is a new calculation of diffraction loss which will apply when the light-emitting layer is surrounded by light-absorbing material.

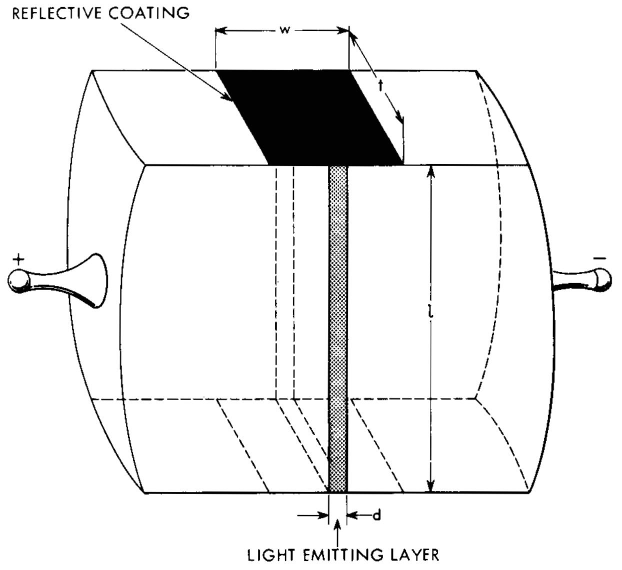
Figure 1 is a diagrammatic sketch of the sort of structure we discuss. It is a rectangular semiconducting crystal with holes and electrons being injected into opposite faces. Within the crystal there is an active layer of thickness d which emits light when a current is passed through the device. The top and bottom surfaces, separated by a distance, l, have reflecting strips of width w. For analytic purposes the dimension t is assumed to be sufficiently great that we may neglect diffraction loss in this direction.

When the device is excited by passing a current through it, there will be a gain per unit length for a light wave in the active region because of stimulated emission. This gain will be proportional to the rate of spontaneous emission. The light wave will simultaneously suffer a loss that is due to reabsorption or scattering by the crystal plus losses that are due to transmission through the reflecting strips and diffraction towards the sides of the crystal. When the gain is sufficient to make up for all of these losses, the device will be at threshold.

Gain from stimulated emission

The gain per unit length is proportional to the rate of spontaneous emission because the probability of stimulated emission into a single electromagnetic

Figure 1 **Diagrammatic sketch of injection laser.**



---

*This work was done under contract with the United States Army Signal Supply Agency (Contract DA 36-039 SC-90711).

† This paper is based upon an internal IBM Research Report, RC-776, August 28, 1962.

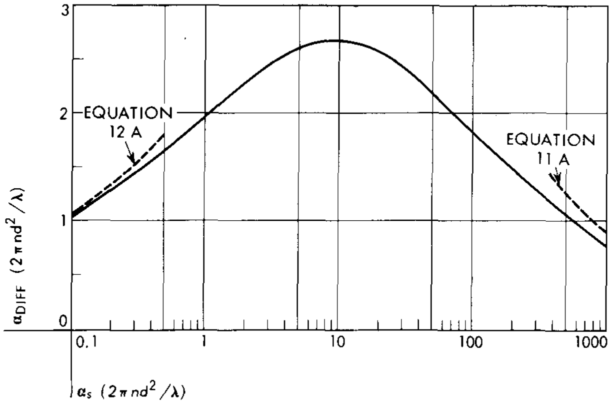


Figure 2 **Absorption coefficient due to diffraction vs absorption coefficient in surrounding material.**

mode equals the probability of spontaneous emission into that mode times the number of quanta in the mode. When we use the usual formula for the number of modes per unit volume per unit frequency interval we find the gain per unit length, g, is given by[4]

$$g = \frac{\lambda^2}{8\pi n^2} R \,, \tag{1}$$

where

R is the number of quanta spontaneously emitted per unit time per unit volume per unit frequency interval (assumed to be equal for all directions of propagation and both polarizations)

λ is the vacuum wavelength of the radiation

n is the index of refraction of the material.

The rate of spontaneous radiation, R, can be expressed as

$$R = \frac{j\eta}{ed\Delta\nu}, \tag{2}$$

where

j is the current density in the device

η is the quantum efficiency (average number of radiation quanta per injected carrier)

e is the electronic charge

d is the thickness of active region

$\Delta\nu$ is the linewidth of the spontaneously emitted radiation whose exact definition is implied by

$$g_{\max}\Delta\nu = \int g(\nu)d\nu = 1 \,, \tag{3}$$

where $g(\nu)$ is the normalized line shape function and $g_{\max}$ is its maximum value.

Losses of the system

The threshold condition is obtained by setting the gain of Eq. (1) equal to the sum of all losses of the radiation wave. One such loss is the absorption or scattering of light by the material. If the lower state of the optical transition is populated it will lead to an absorption due to the inverse of the emission process. Of course other impurities, crystal defects and free carriers can give additional absorption or scattering.

In addition to this bulk absorption there is an effective absorption per unit length, which is due to the imperfect resonant structure. The finite transmission through the reflecting coatings is equivalent to an absorption of $\alpha_T = T/l$, where T is the transmission of the coatings and l, as above, is the distance between reflectors.

Because of diffraction, all of the light reflected from one strip is not incident upon the other one. The loss due to this effect for the most favorable mode has been calculated by Fox and Li under the assumption that active layer thickness, d, and reflector width, w, are equal.[5] Their result for small values of the parameter, $l\lambda/nd^2$, is:

$$\alpha_{\mathrm{Diff}} = \frac{0.35}{l}\left[\frac{l\lambda}{nd^2}\right]^{3/2} . \tag{4}$$

The most efficient injection light source at present is a GaAs diode, which probably has a very small thickness, d, of its light-emitting layer. It is therefore of interest to consider how this diffraction loss may be decreased. One remedy which works in principle is to use confocal reflectors which could, if necessary, be separate from the crystal. The modes for such a structure have been obtained analytically by Boyd and Gordon.[6] They show the field strength of the central mode varies with transverse distance roughly as a Gaussian with a width of $\sqrt{l\lambda/2\pi}$ and may have negligible diffraction loss. To obtain modes undistorted by absorption in the surrounding material this mode width should not be much larger than d, the thickness of the active region, and this may require unrealistically small values of the length l.

For thin, actively emitting layers and broad reflecting strips, it is quite probable that the transverse extent of the lowest loss mode would be determined by the light absorption of the non-emitting material surrounding the active layer. The diffraction loss for this case is determined in the Appendix. The result is insensitive to the absorption if the extinction coefficient in the inactive material is of the order of $10\lambda/2\pi nd^2$. For this case our result is

$$\alpha_{\mathrm{Diff}} = 0.42\lambda/nd^2 \,. \tag{5}$$

Figure 2 gives α_{Diff} as a function of the absorption coefficient in the surrounding material, α_s. The applicability of this calculation depends upon the amplitude of the mode being small at the edge of the reflecting strips which will be the case if $w \gg d$ and the absorption coefficient is not too small. The final paragraph of the Appendix explains how one may find the amplitude of the wave at the edge of the reflecting strips and thereby determine whether this theory of diffraction loss applies to any particular case.

The threshold relation

By equating the gain due to stimulated emission, Eqs. (1) and (2), to the sum of all losses we find an expression for the threshold current density:

$$j = \frac{8\pi e n^2 d \Delta\nu}{\eta\lambda^2}\left[\alpha_0 + T/l + \alpha_{\mathrm{Diff}}\right]. \tag{6}$$

In a convenient set of units, this becomes

$$j(\mathrm{amp/cm^2}) = 6.3 \times 10^4 \, \frac{n^2 d E^2 \Delta E}{\eta}\left[\alpha_0 + \frac{T}{l} + \alpha_{\mathrm{Diff}}\right], \tag{7}$$

where

E is energy of radiation in electron volts
ΔE is linewidth of spontaneous emission in electron volts (the $\Delta\nu$ of Eq. (3) in energy units)
d is thickness of the light-emitting layer
η is the quantum efficiency
n is the index of refraction
α_0 is absorption coefficient in the light-emitting layer
T is fraction of light transmitted through the reflecting coatings
l is the distance between the reflecting coatings
α_{Diff} is the effective absorption coefficient due to diffraction of light.

For large d (i.e., $d^2 \gg l\lambda/n$) and reflecting strips of width d, Eq. (4) applies; for reflecting strips much wider than d and light absorption in the material surrounding the light emitting layer, the theory of the Appendix applies, the values of α_{Diff} are given in Fig. 2, and Eq. (5) gives an upper bound for α_{Diff} in this case.

Application to gallium arsenide injection lasers

At the present time the values of many of the quantities which enter the expression for threshold current are unknown. We can, however, assume some values for the sake of illustration. At normal incidence an uncoated gallium arsenide crystal has a reflectivity of 30%. A distance of one mm between reflecting surfaces then implies an effective absorption coefficient of 7 cm^{-1}. We now ask what thickness of light emitting layer will yield an equal absorption coefficient due to diffraction, assuming that the conditions for validity of Eq. (5) are met. The result is $d = 1.1 \times 10^{-3}$ cm. For emitting layers thicker than this the reflection loss will be more important and conversely thinner layers will make the diffraction loss dominate. Very little is known about bulk absorption and scattering that is due to incomplete inversion of energy level populations or crystal imperfections. If we assume a diffraction loss equal to the reflection loss the computed threshold is $j = 830$ amp/cm^2, where we have used the values $d = 1.1 \times 10^{-3}$ cm, $n = 4$, $E = 1.47$ ev, $\Delta E = 0.025$ and $\eta = 1$ in Eq. (7). This threshold is only a factor of about 10 less than that observed in Ref. 2, showing that our assumptions are not completely wide of the mark.

Alternatively one can ask what thickness of light-emitting layer would give a sufficiently large diffraction loss to account for the threshold reported in Ref. 2. This value is

$$d \cong 0.6 \times 10^{-4}\ \mathrm{cm}.$$

It is more likely, however, that other losses, particularly absorption due to incomplete inversion, are important in present devices at liquid nitrogen temperature.

Acknowledgment

It is a pleasure to acknowledge many helpful conversations with various members of this laboratory, including the authors of References 1 to 3 as well as R. W. Landauer, R. W. Keyes, A. E. Michel and E. V. Walker. We are also grateful to Frank Stern and Marshall Nathan for their critical reading of the manuscript.

Appendix

We assume an infinite slab of light-emitting material with boundaries at $x = \pm d/2$ surrounded by light-absorbing material having an extinction coefficient α_s. The real part of the index of refraction, n, is assumed to be equal in both materials. We seek to determine the minimum value of gain per unit length, g, in the light-emitting material necessary to support a uniformly propagating light wave in this structure. This wave will have a large amplitude in the light-emitting material and an exponentially damped amplitude in the absorbing material. Its propagation vector will lie in the plane of the slab and we can take it to be of the form

$$\Phi(x, z, t) = \phi(x)\exp[i(kz - \omega t)], \tag{1A}$$

where k and ω are real numbers. This function must satisfy the wave equation

$$\left[\frac{n^2}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{n}{c}\, g\, \frac{\partial}{\partial t}\right]\Phi = 0 \tag{2A}$$

for $|x| < d/2$ and the same equation with g replaced by $-\alpha_s$ for $|x| > d/2$. Hence $\phi(x)$ satisfies

$$\left[k^2 - \frac{n^2\omega^2}{c^2} - \frac{in\omega g}{c} - \frac{\partial^2}{\partial x^2}\right]\phi(x) = 0\,. \tag{3A}$$

The solutions of this equation which are even in x and vanish for large x are

$$\phi(x) = \left\{\begin{array}{ll} \cos ax & \text{for} \quad |x| < d/2 \\ \phi_d e^{-b|x|} & \text{for} \quad |x| > d/2 \end{array}\right\}, \tag{4A}$$

where a and b are complex numbers. By substituting into Eq. (3A) we find

$$k^2 - \frac{n^2\omega^2}{c^2} - \frac{in\omega g}{c} + a^2 = 0$$

$$k^2 - \frac{n^2\omega^2}{c^2} + \frac{in\omega\alpha_s}{c} - b^2 = 0\,. \tag{5A}$$

The condition that the logarithmic derivative of $\phi(x)$ be continuous at $x = \pm d/2$ is that

$$a\tan(ad/2) = b\,. \tag{6A}$$

The difference of the two equations (5A) gives

$$a^2 + b^2 = \frac{in\omega}{c}(g + \alpha_s)\,. \tag{7A}$$

By eliminating b between the last two equations,

$$a^2 = \frac{in\omega}{c}(g + \alpha_s)\cos^2(ad/2)\,. \tag{8A}$$

The complex phase of this equation depends only on ad, and we use this fact to derive a relation between the real and imaginary parts of ad:*

$$\frac{u - v}{u + v} = \tan u \tanh v\,, \tag{9A}$$

where

$u = 1/2d \operatorname{Re} a$

$v = 1/2d \operatorname{Im} a\,.$

Solutions of (9A) with u in the range $0 < u < \pi/2$ correspond to nodeless solutions of the wave equation (3A) and therefore give us the solutions of least loss. Pairs of numbers, (u, v), satisfying (9A) are easily obtained by successive approximation, and from each such pair we find corresponding values of g and α_s from

$$g = 8uvg_0$$

$$g + \alpha_s = 2\left[\frac{u + v}{\cos u \cosh v}\right]^2 g_0\,, \quad \text{where} \tag{10A}$$

$$g_0 = \frac{c}{n\omega d^2} = \frac{\lambda}{2\pi n d^2}\,.$$

The gain, g, is the effective absorption coefficient due to diffraction, α_{Diff}, discussed in the text. It turns out to be slowly varying with α_s, and we therefore present our results in Fig. 2 by plotting g/g_0 versus α_s/g_0 on a logarithmic scale. The maximum value of g/g_0 is 2.66, and this gives us Eq. (5) of the text. The gain required goes to zero for large α_s as well as small, because as $\alpha_s \to \infty$ the amplitude of the solution in the absorbing material goes to zero. The asymptotic expressions for large and small α_s are

$$\text{as} \quad \alpha_s/g_0 \to \infty; \quad g/g_0 \to 2\pi^2\sqrt{\frac{2g_0}{\alpha_s}} \tag{11A}$$

$$\text{as} \quad \alpha_s/g_0 \to 0; \quad g/g_0 \to 2\sqrt[3]{\frac{3\alpha_s}{2g_0}}\,. \tag{12A}$$

Finally, we discuss the conditions for the validity of the above theory of diffraction loss. One condition arises from a low frequency cutoff as in a waveguide which occurs when k^2 in Eq. (5A) has a negative value. From the first equation of (5A) we find that this requires that

$$\frac{n^2\omega^2}{c^2} - \operatorname{Re}\alpha^2 > 0\,.$$

For the variable u in the range $(0, \pi/2)\operatorname{Re}\alpha^2$ cannot exceed π^2/d^2 and thus we get the simpler sufficient but not the necessary condition $d > \lambda/2n$. In an actual device we must also require that the wave not be limited in the x-direction by the finite extent of the reflectors, and this requires that the reflector width, d, be much greater than $1/b$.

References and footnotes

1. M. I. Nathan, W. P. Dumke, G. Burns, F. H. Dill, Jr., and G. J. Lasher, *Appl. Phys. Letters* **1**, 62 (1962).
2. G. Burns, R. A. Laff, S. E. Blum, F. H. Dill, Jr., and M. I. Nathan, *IBM Journal*, this issue p. 62.
3. R. A. Laff, W. P. Dumke, F. H. Dill, Jr., and G. Burns, *IBM Journal*, this issue, p. 63.
4. If N is the number of quanta in a given electromagnetic mode, and $\frac{dN}{dt}$ its time rate of change due to stimulated emission, then the gain is $g = \frac{n}{cN}\frac{dN}{dt}$. The statement in the text about the probability of stimulated emission into a single mode implies $\frac{1}{N}\frac{dN}{dt} = R/n_\nu$, where $n_\nu = \frac{8\pi n^3}{\lambda^2 c}$ is the number of electromagnetic modes per unit frequency interval per unit volume.
5. A. G. Fox and T. Li, *Bell System Tech. J.* **40**, 453 (1961).
6. G. D. Boyd and J. P. Gordon, *Bell System Tech. J.* **40**, 489 (1961).

Received October 31, 1962

---

* In fact we multiply Eq. (8A) by $-i$, take the square root and equate the ratio of imaginary to real parts of both sides of the resulting equation.